

# Tipping points in grounding zone melting via seawater intrusions

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**OXFORD**



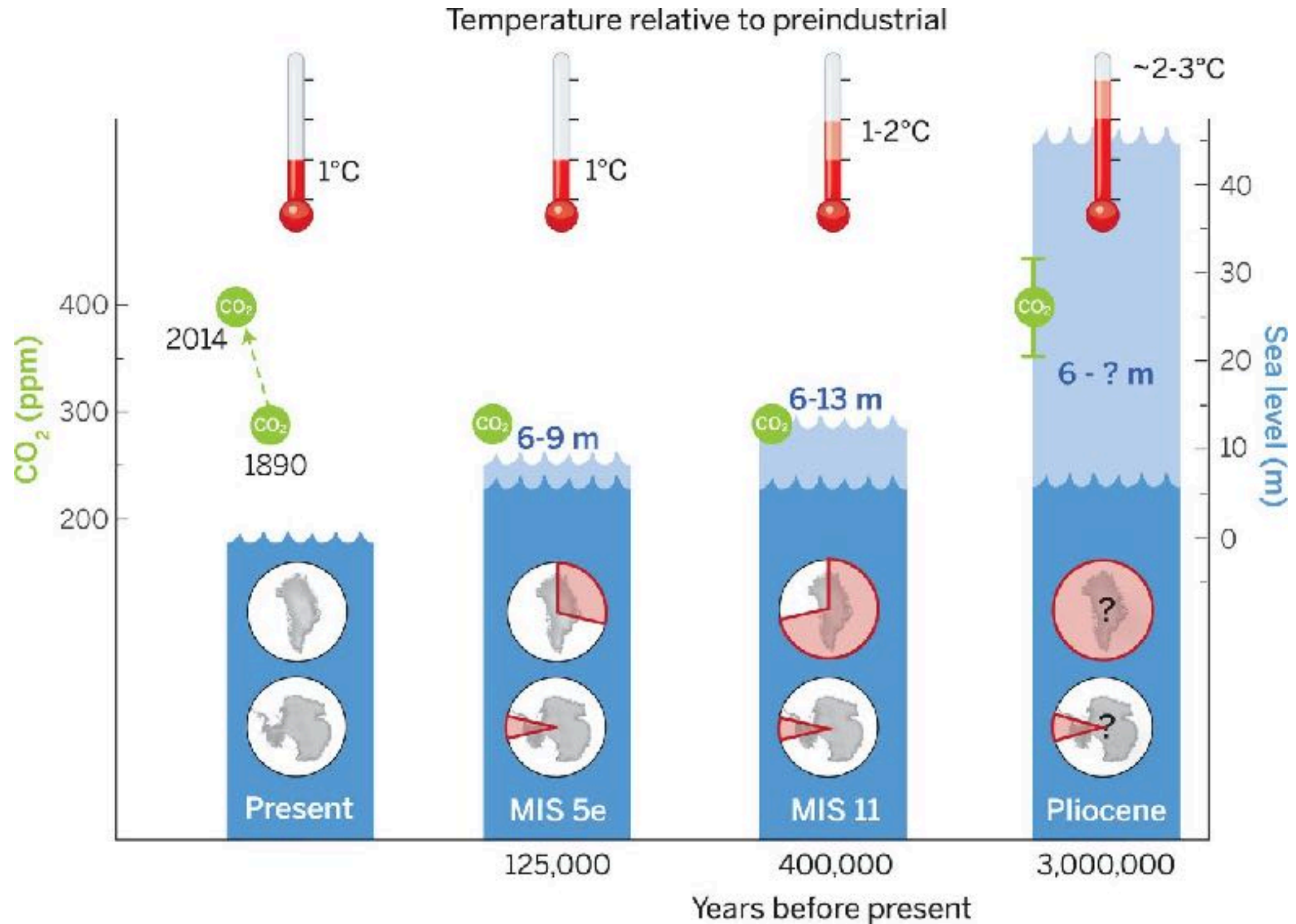
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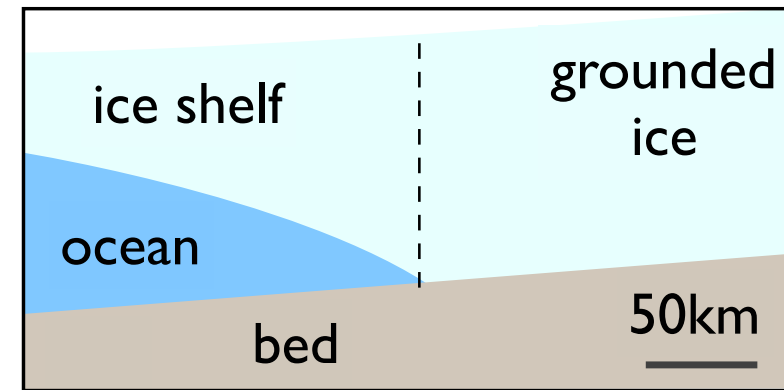
# Sea levels have been much higher than today with similar CO<sub>2</sub> and temperatures



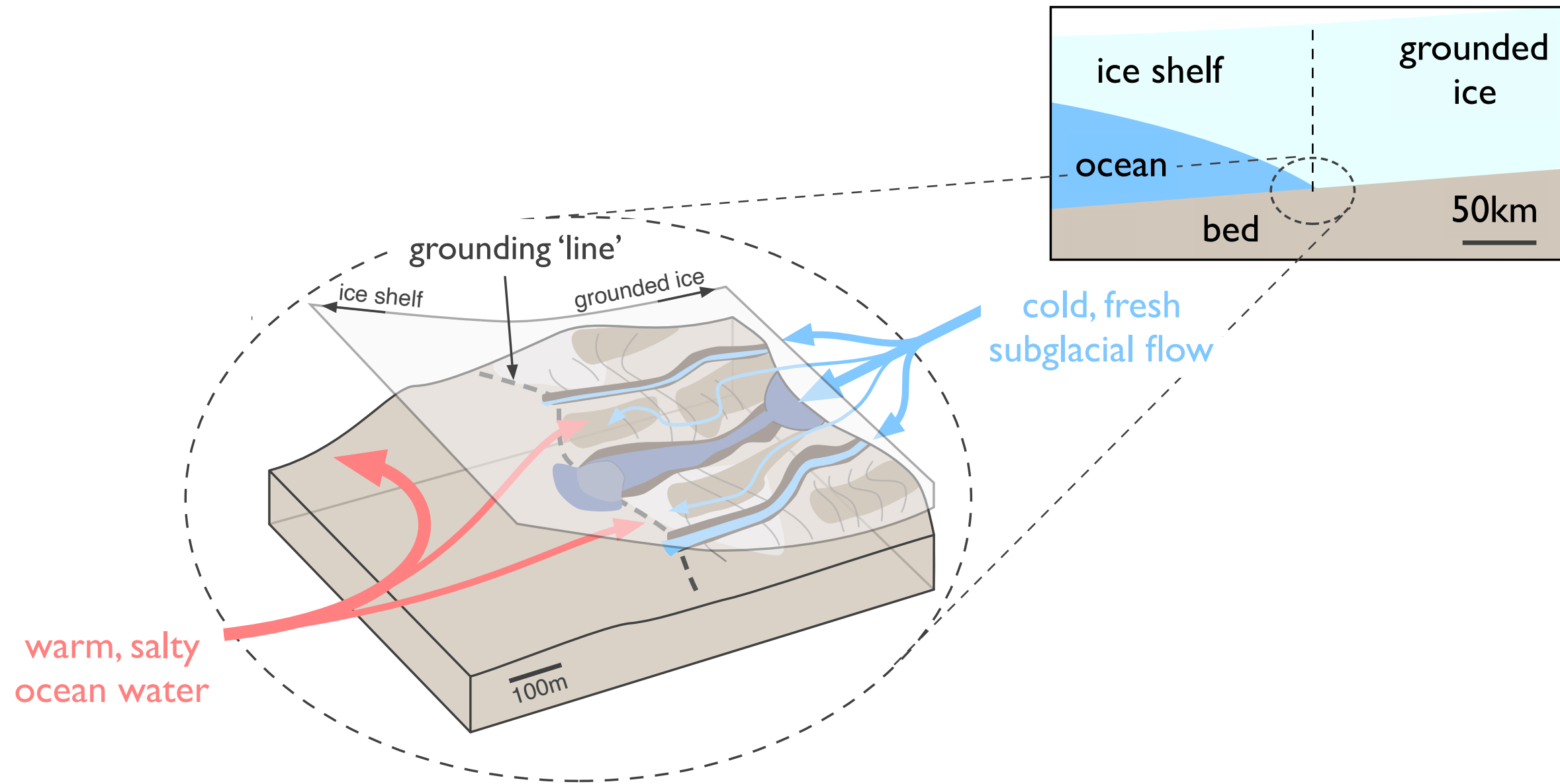




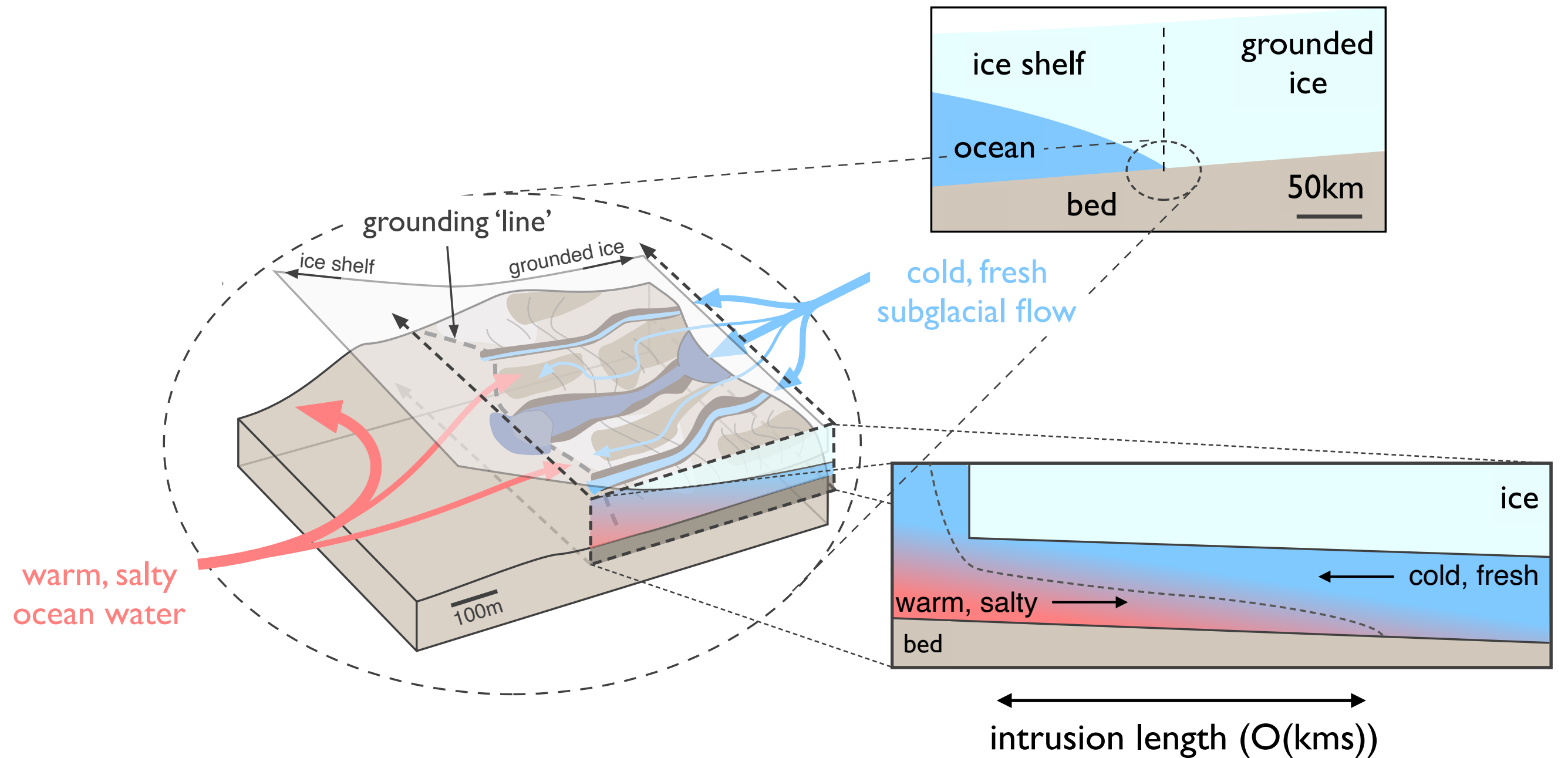
# Grounding zone melt boosts ice sheet sensitivity



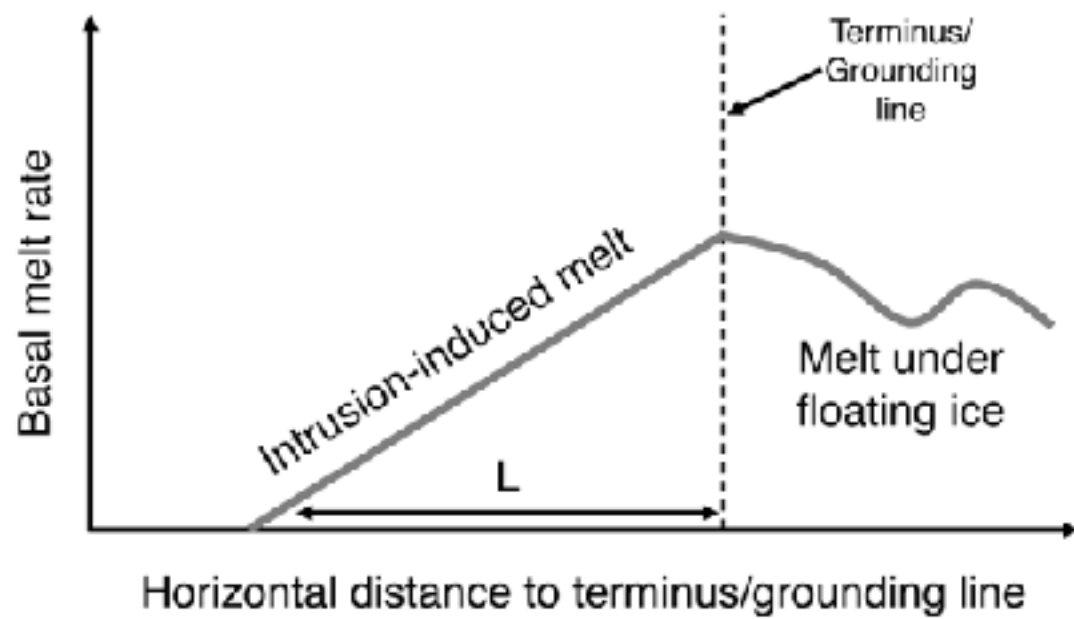
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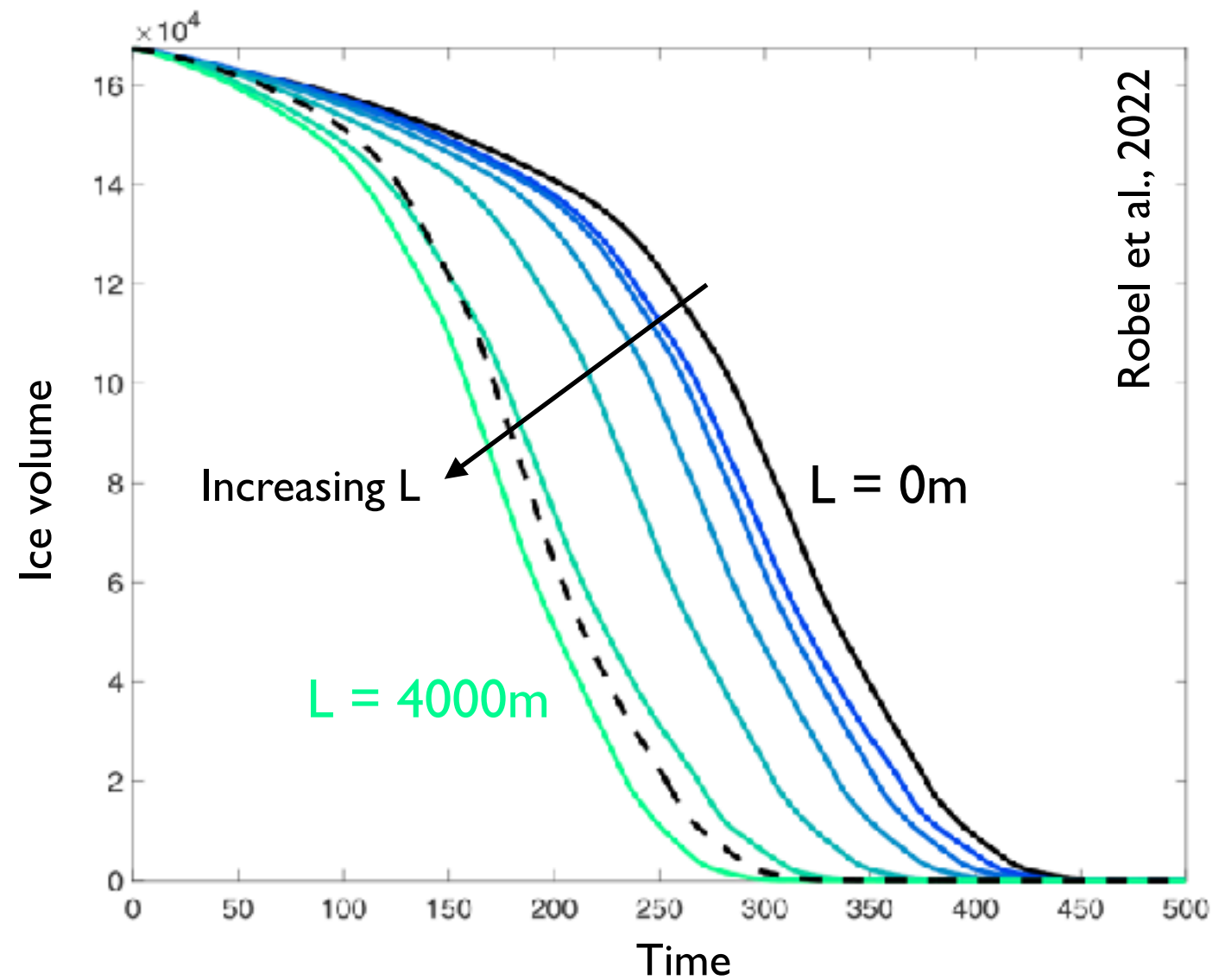


# Grounding zone melt boosts ice sheet sensitivity



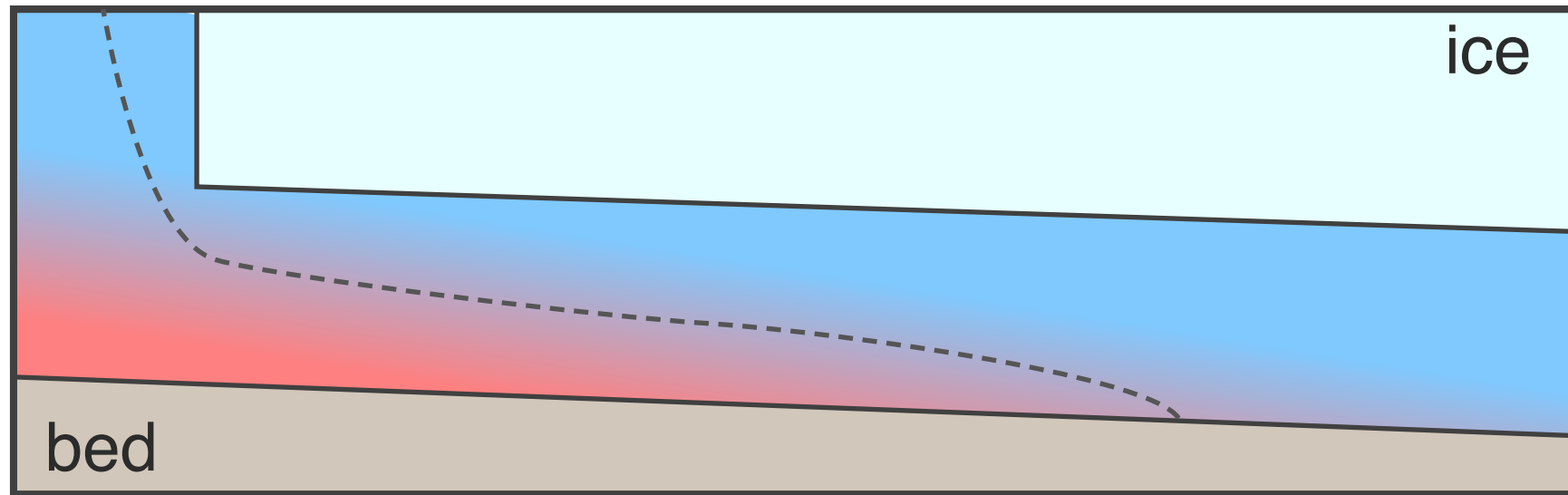
Robel et al., 2022

**Significant seawater intrusion has dramatic consequences for ice dynamics**



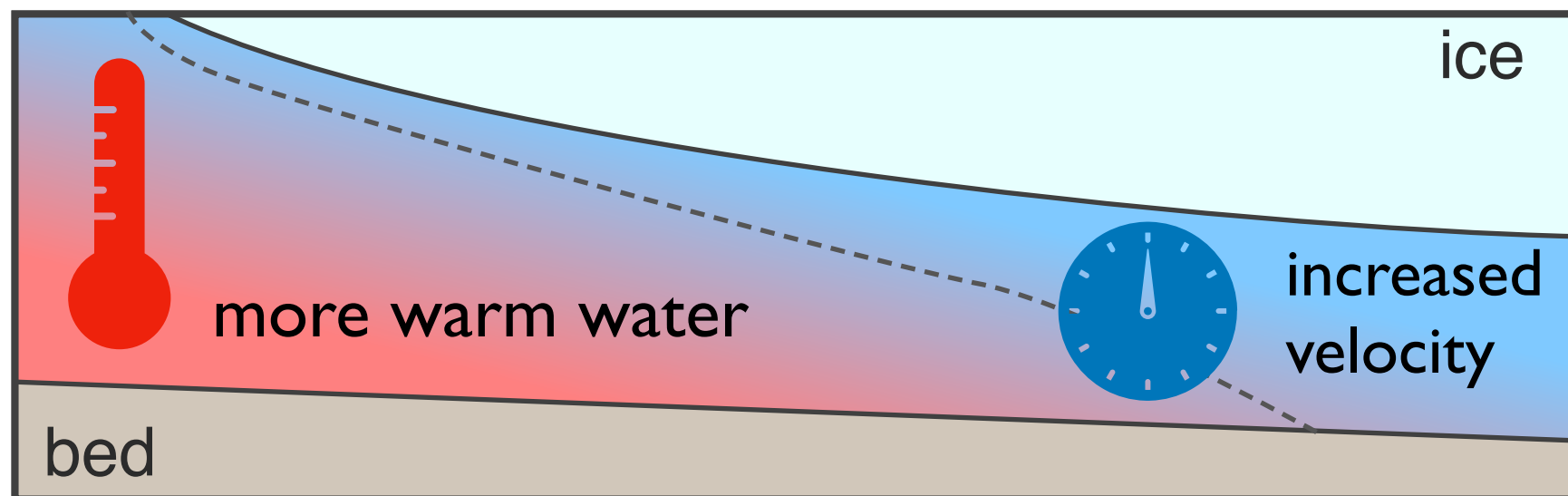
Robel et al., 2022

# Melt feedbacks on seawater intrusions

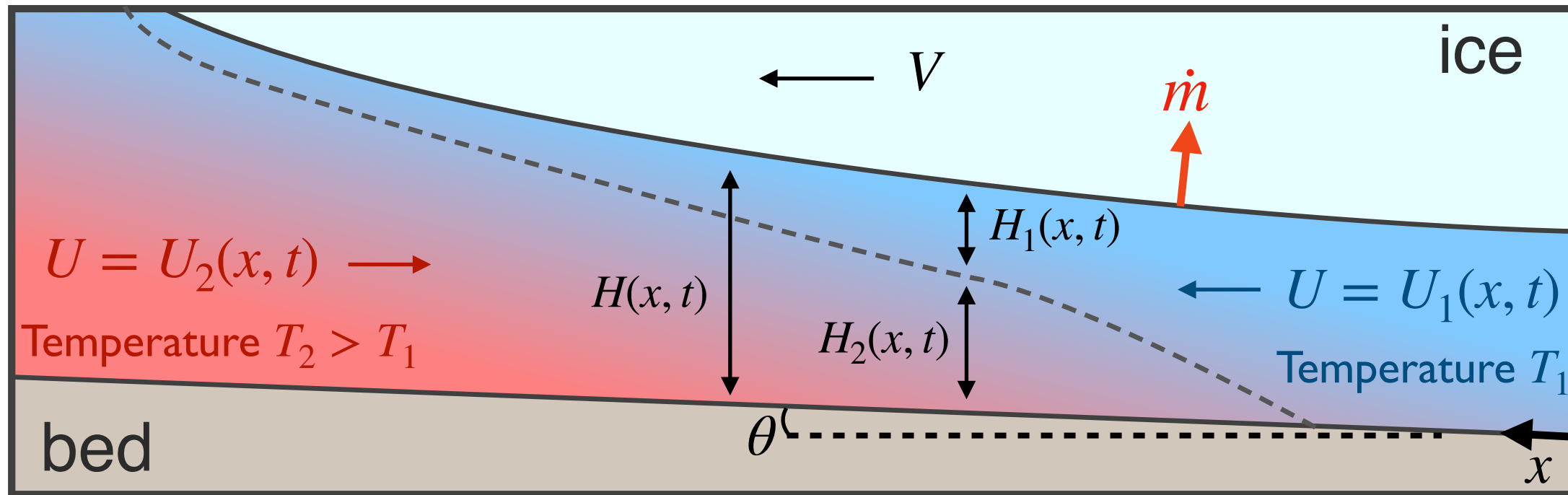


melt response

A vertical arrow pointing downwards from the top diagram to the bottom diagram, indicating the progression of the melt response.

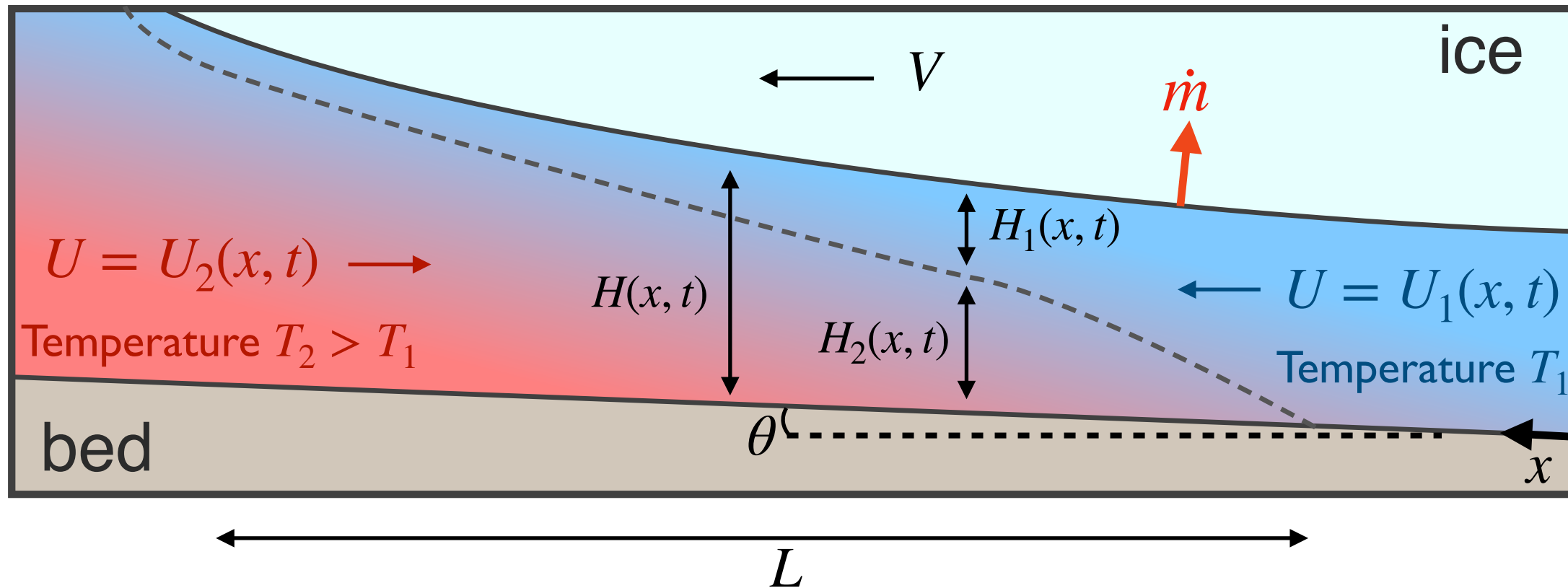






$$H = H_\infty$$

$$U = U_\infty$$



$$H = H_\infty$$

$$U = U_\infty$$

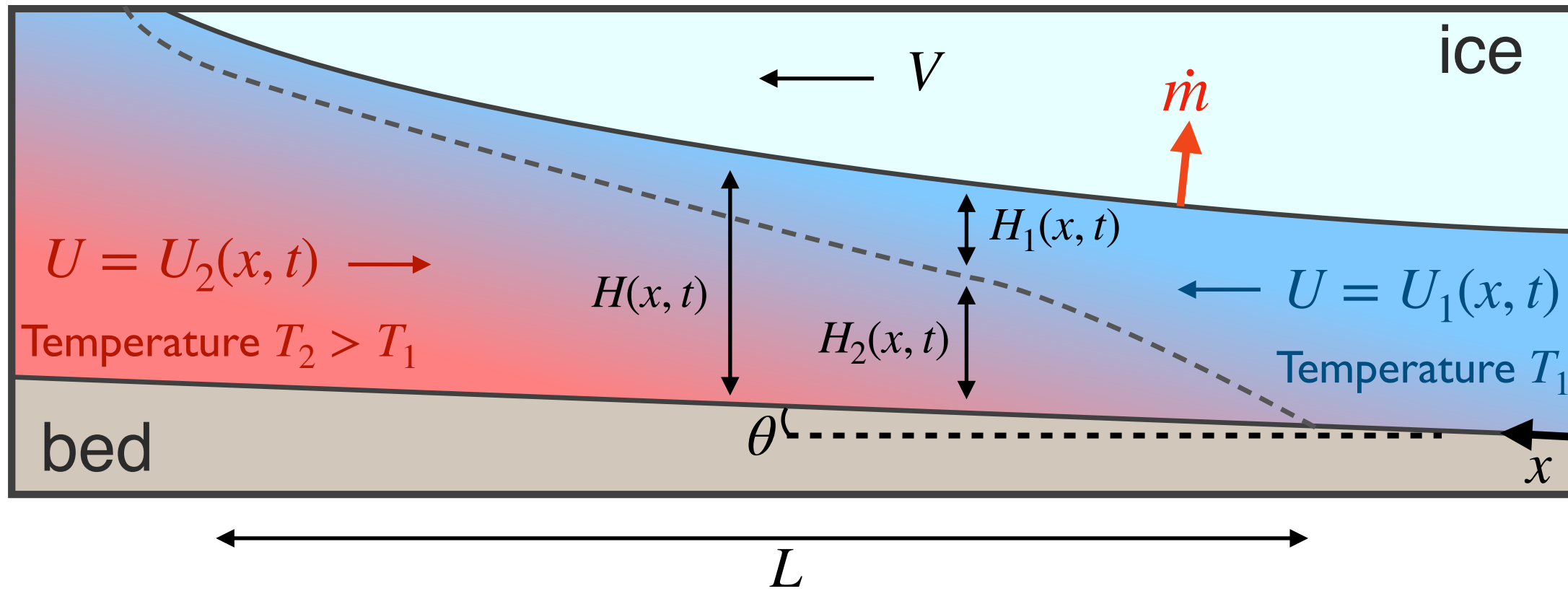
Momentum Conservation:

inertia      barotropic pressure gradient      interfacial drag      wall drag      gravitational driving

$$\frac{\partial U_1}{\partial t} + U_1 \frac{\partial U_1}{\partial x} + \frac{1}{\rho} \frac{\partial P}{\partial x} + \frac{C_i |U_1 - U_2| (U_1 - U_2)}{H_1} + \frac{C_d U_1^2}{H_1} = 0$$

$$\frac{\partial U_2}{\partial t} + U_2 \frac{\partial U_2}{\partial x} + \frac{1}{\rho} \frac{\partial P}{\partial x} - \frac{C_i |U_1 - U_2| (U_1 - U_2)}{H_2} + \frac{C_d U_2^2}{H_2} + g' \left( \frac{\partial H_2}{\partial x} + \tan \theta \right) = 0$$

$$(\text{Fr}^2 - 1) \frac{\partial H_1}{\partial x} = \text{Fr}^2 \left( C_d + C_i \frac{H}{H - H_1} \right) - \left( \tan \theta + \frac{\partial H}{\partial x} \right)$$

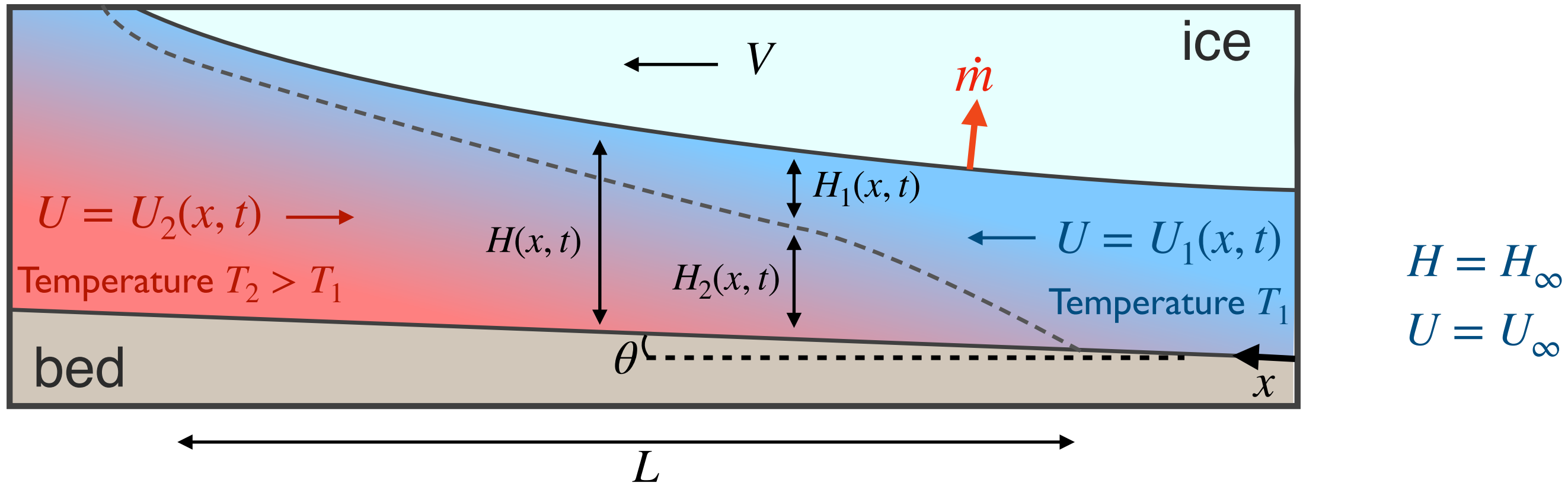


$$H = H_\infty$$

$$U = U_\infty$$

Momentum Conservation:

$$(\text{Fr}^2 - 1) \frac{\partial H_1}{\partial x} = \text{Fr}^2 \left( C_d + C_i \frac{H}{H - H_1} \right) - \left( \tan \theta + \frac{\partial H}{\partial x} \right)$$



Momentum Conservation:

$$(\text{Fr}^2 - 1) \frac{\partial H_1}{\partial x} = \text{Fr}^2 \left( C_d + C_i \frac{H}{H - H_1} \right) - \left( \tan \theta + \frac{\partial H}{\partial x} \right)$$

Melting:  $\dot{m} = \frac{\text{St}C}{L} u^* \Delta T$

thermal driving

boundary layer velocity  $u^* = U_1$

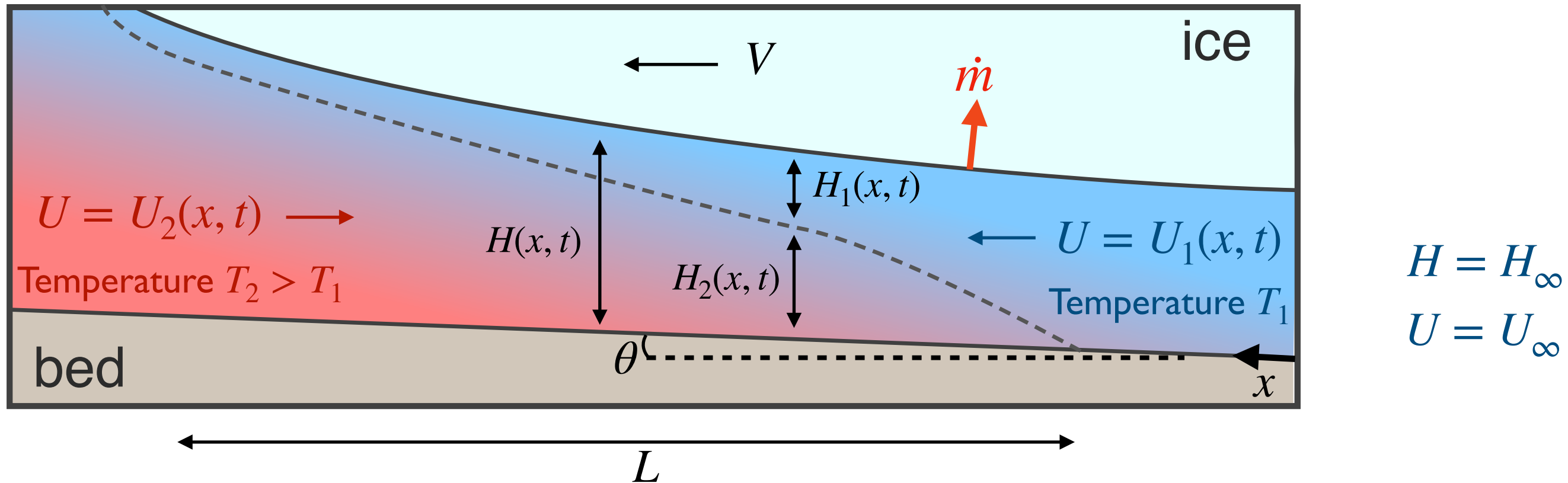
$$\Delta T = T - T_f$$

$$T = \frac{H_1}{H} T_1 + \left( 1 - \frac{H_1}{H} \right) T_2$$

$T_f$ : local freezing point

$$\dot{m} = \frac{\text{St}C}{L} U_1 \left[ \frac{H_1}{H} T_1 + \left( 1 - \frac{H_1}{H} \right) T_2 \right]$$



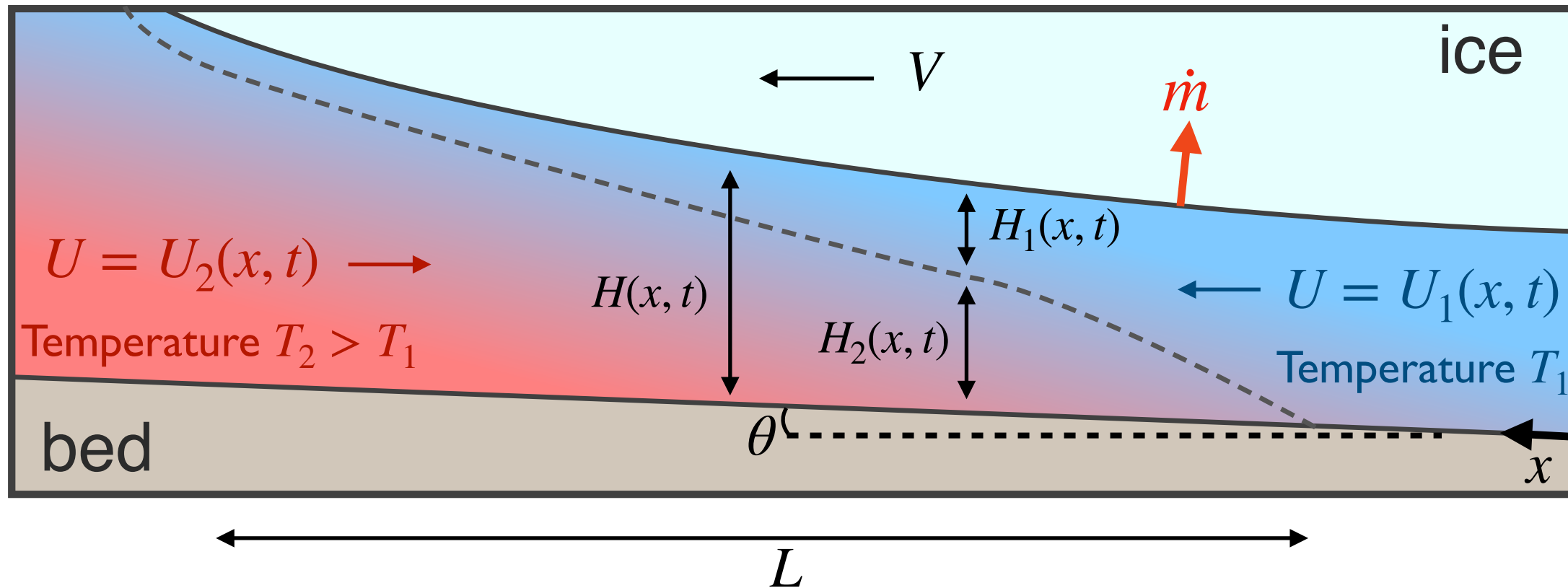


Momentum Conservation:

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Melting:

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Momentum Conservation:

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Melting:

$$\dot{m} = \frac{\text{St}C}{L} U_1 \left[ \frac{H_1}{H} T_1 + \left( 1 - \frac{H_1}{H} \right) \right]$$

Kinematic Condition:

$$\frac{\partial H}{\partial t} + V \frac{\partial H}{\partial x} = \dot{m}$$

Momentum conservation:

$$\left( \frac{F^2}{h_1^3} - 1 \right) \frac{\partial h_1}{\partial x} = \frac{F^2}{h_1^3} \left( 1 + C \frac{h}{h - h_1} \right) - \left( S + \frac{\partial h}{\partial x} \right)$$

Melting + kinematic:

$$\frac{\partial h}{\partial t} + \frac{1}{M} \frac{\partial h}{\partial x} = \frac{1}{h_1} \left( 1 - \frac{h_1}{h} \right)$$

$$S = \frac{\tan \theta}{C_d}$$

dimensionless bed slope

$$F = \frac{U_\infty}{\sqrt{g' H_\infty}}$$

upstream Froude number

$$C = \frac{C_i}{C_d}$$

dimensionless drag

$$M = \frac{u_\infty}{V} \frac{St}{c_d} \frac{T_2 - T_1}{L/c}$$

dimensionless melt

+ boundary condition:  $h_1^{3/2} = F^{2/3}$  at  $x = 0$

$$T_2 = 1.9^\circ\text{C} \quad (M = 0.38)$$

t = 0.0



70m (no melt feedback, Robel et al. 2020)

110m with melt feedback

'bounded intrusion'

$$M < M_c$$

$$T_2 = 2^\circ\text{C} \quad (M = 0.4)$$

t = 0.0



70m

$L \rightarrow \infty$

'unbounded intrusion'

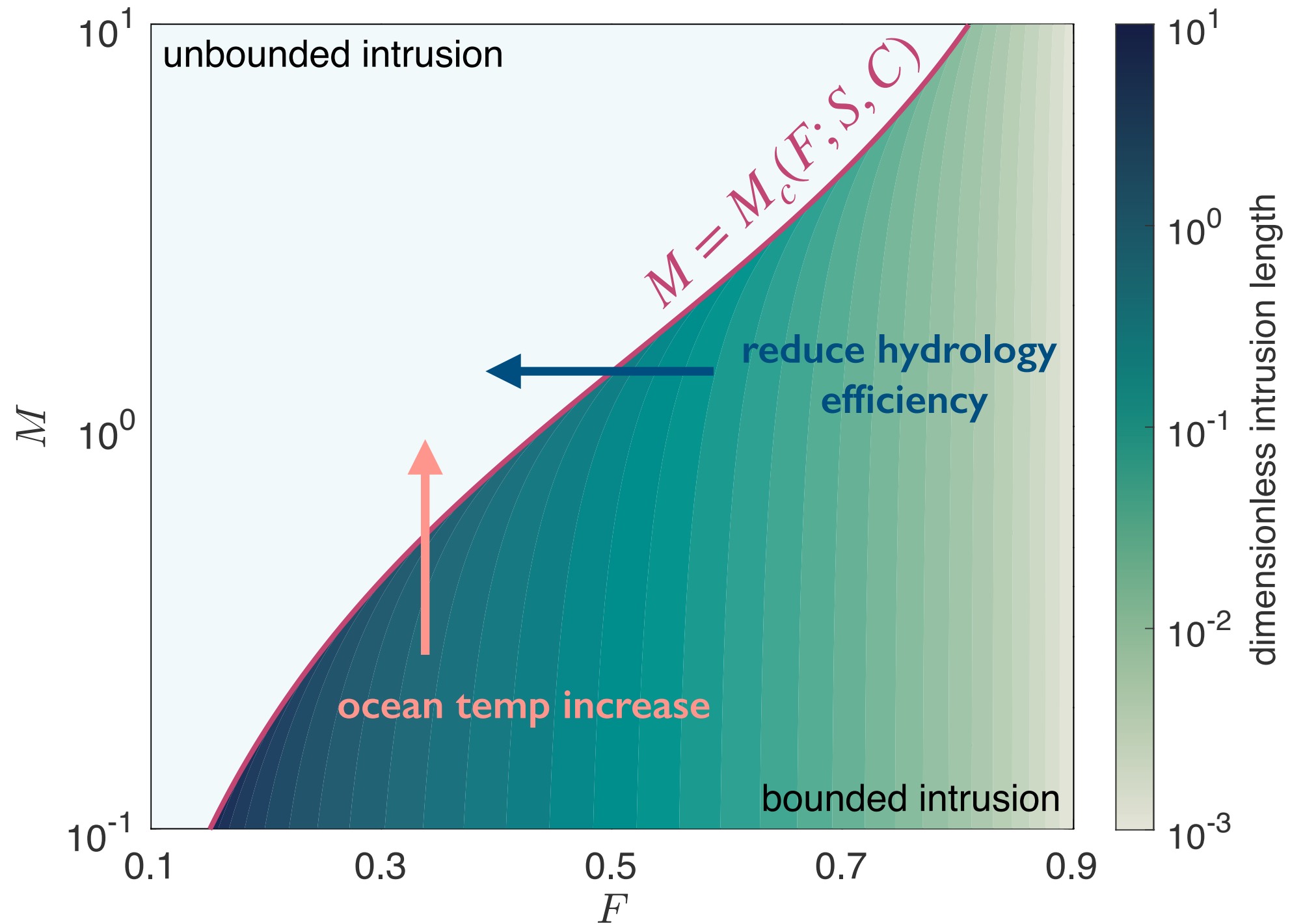
$$M > M_c$$

**A small change in ocean temperature leads to a large response in grounding zone melting, with significant implications for ice dynamics**

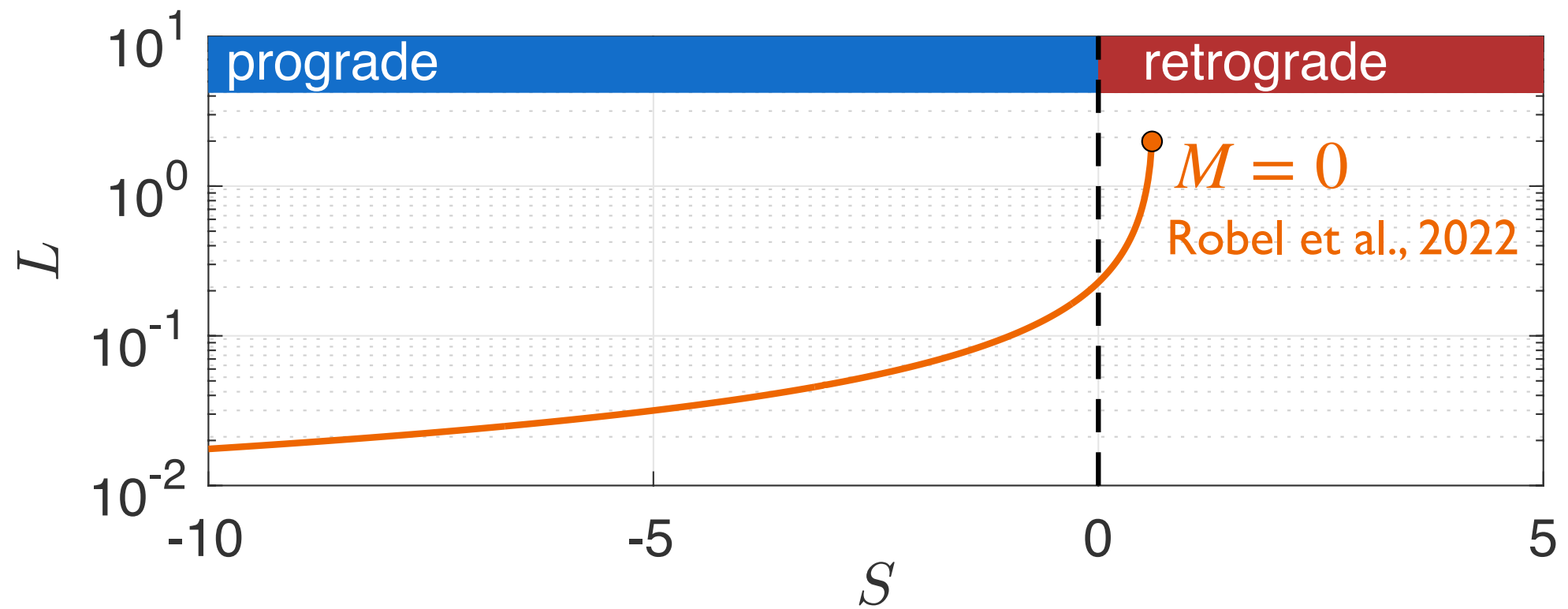
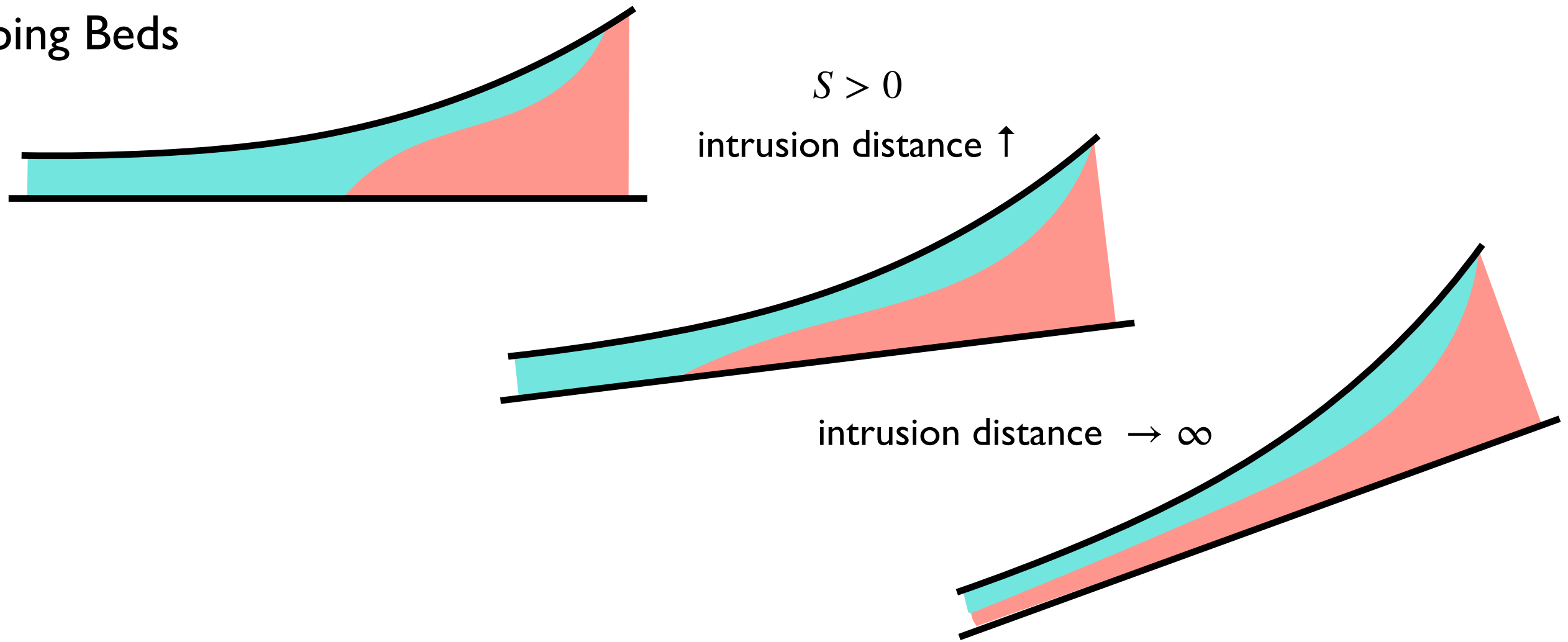


$$M = \frac{u_\infty}{V} \frac{St}{c_d} \frac{T_2 - T_1}{L/c}$$

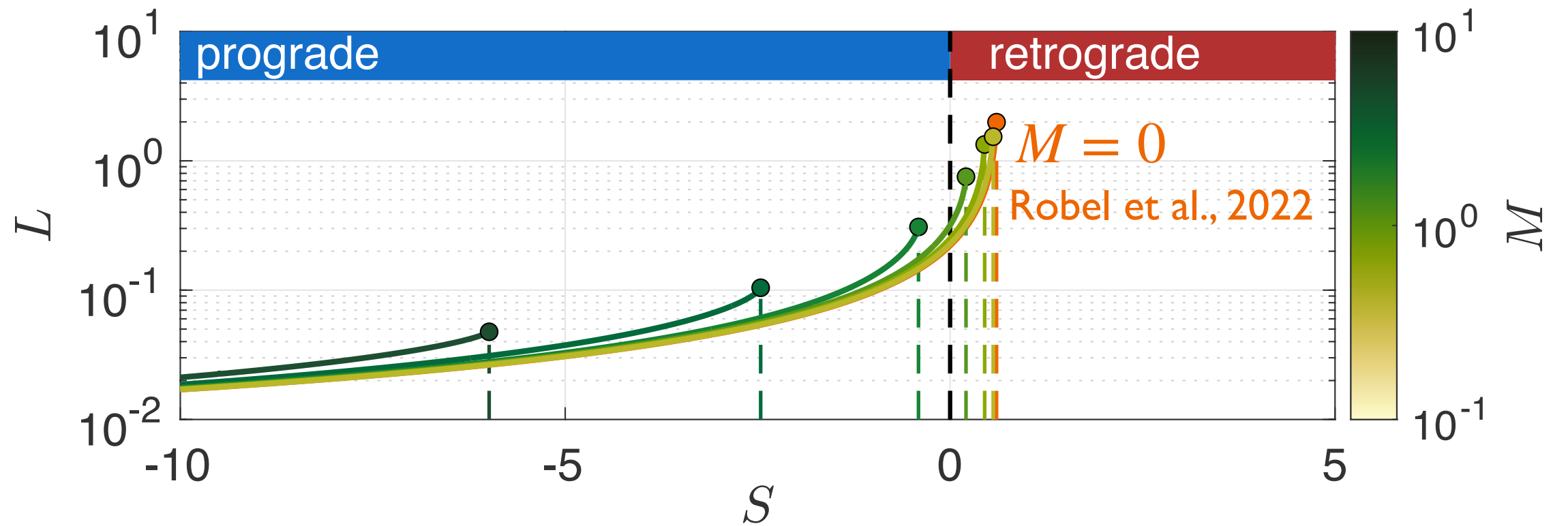
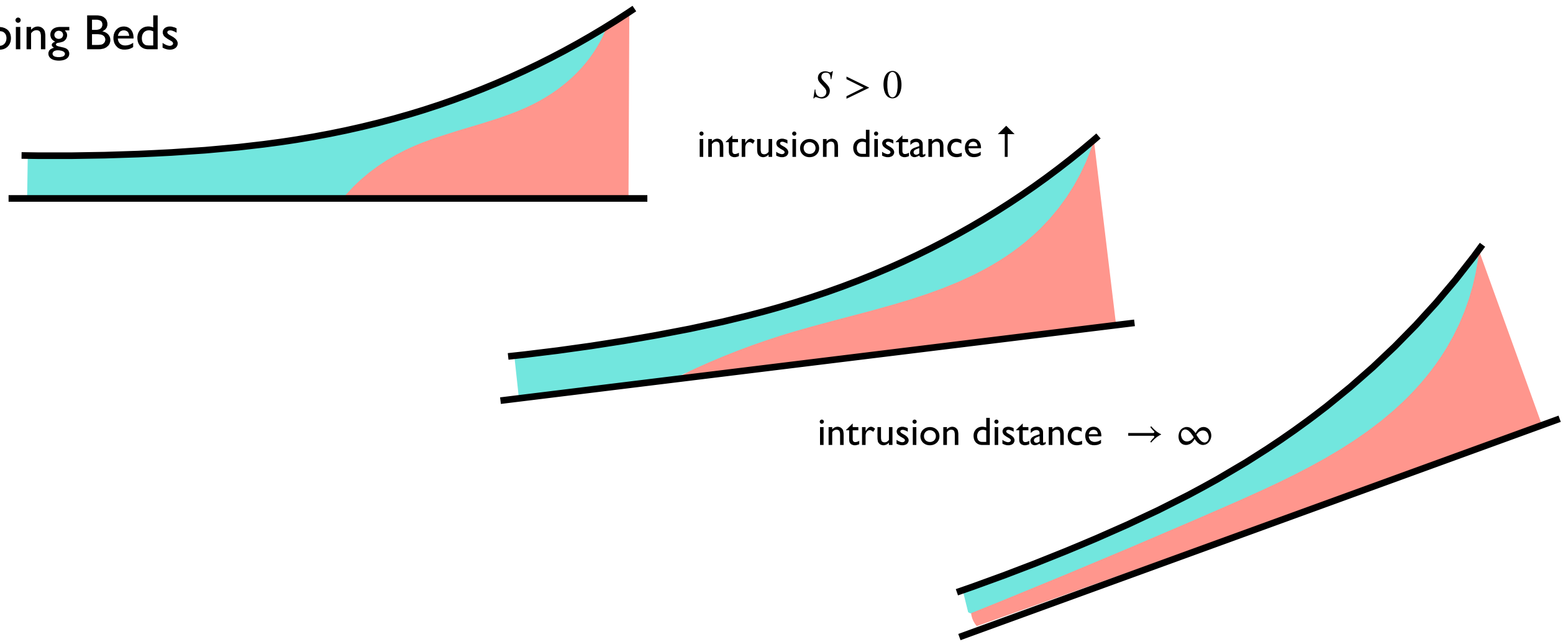
$V$ : ice velocity  
 $u_\infty$ : upstream meltwater velocity  
 $T_2 - T_1$ : ocean forcing



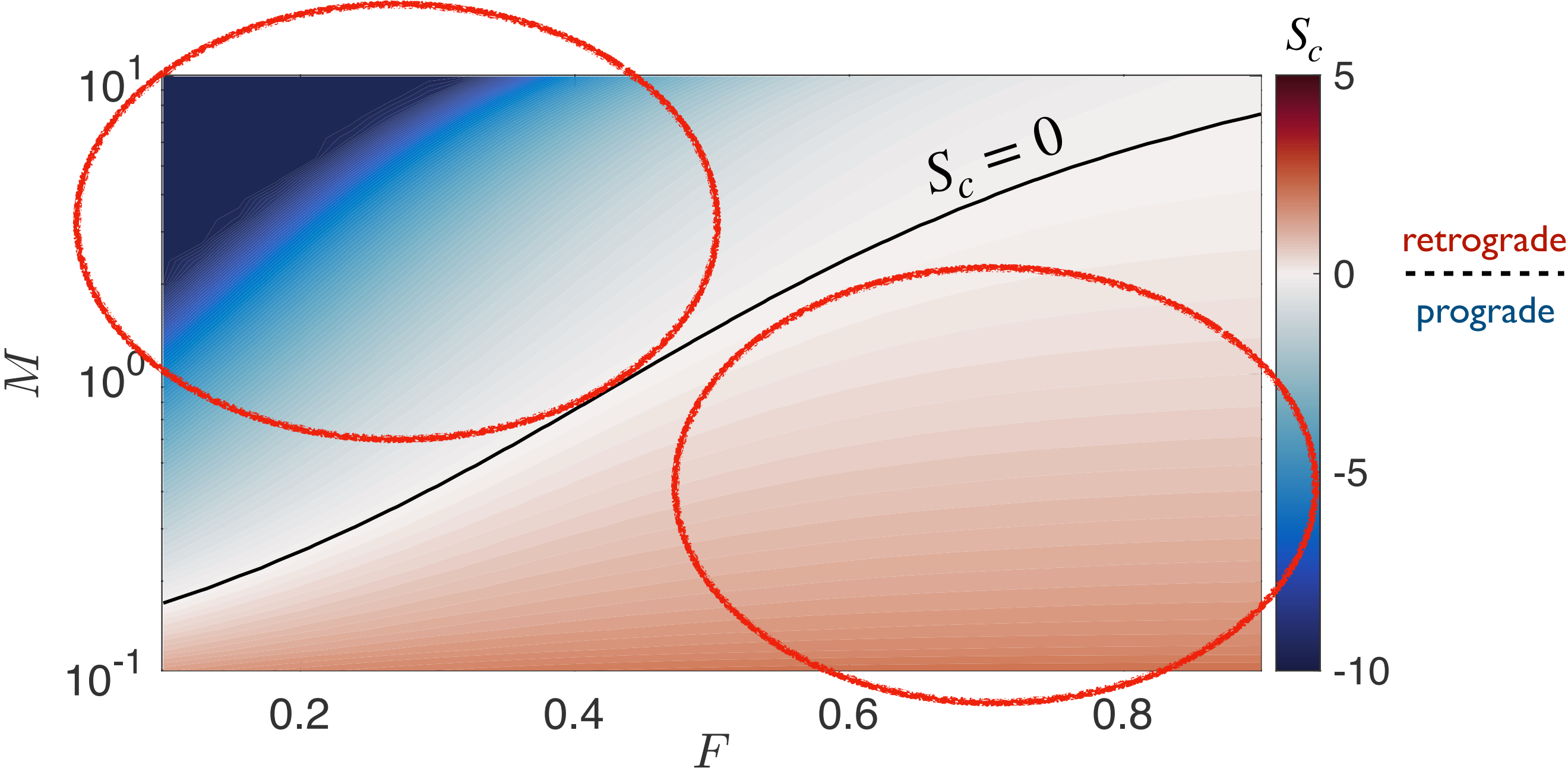
# Sloping Beds



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prograde bedslopes can also have unbounded intrusion - less stable than we think?



melt feedback makes unbounded intrusion easier on retrograde bedslopes - candidate mechanism to explain warm period retreat?



F is poorly constrained, plot as a function of M, S

$$S = \frac{\tan \theta}{C_d}$$

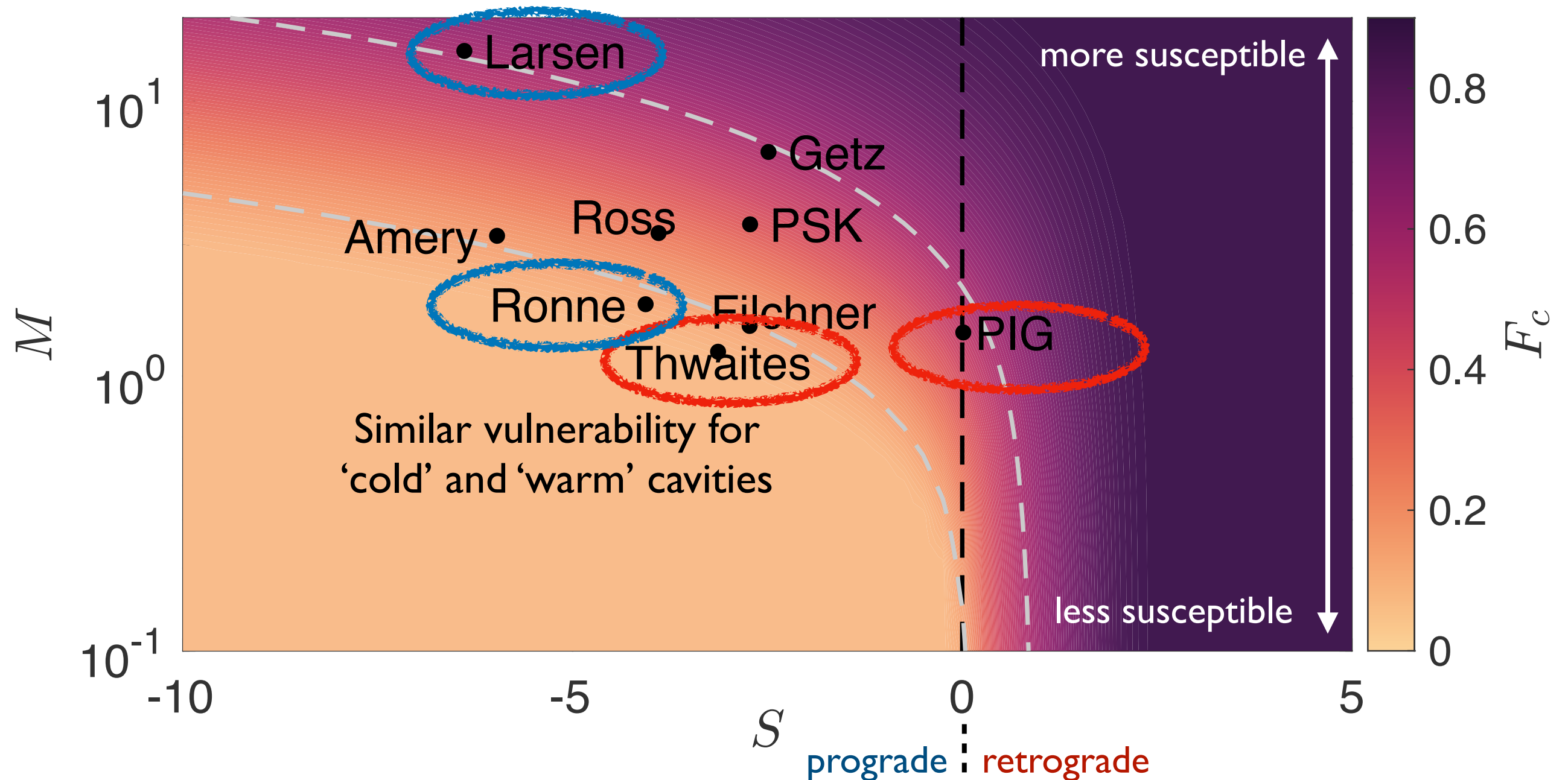
dimensionless bed slope

$$F = \frac{U_\infty}{\sqrt{g' H_\infty}}$$

upstream Froude number

$$M = \frac{u_\infty}{V} \frac{St}{c_d} \frac{T_2 - T_1}{L/c}$$

dimensionless melt



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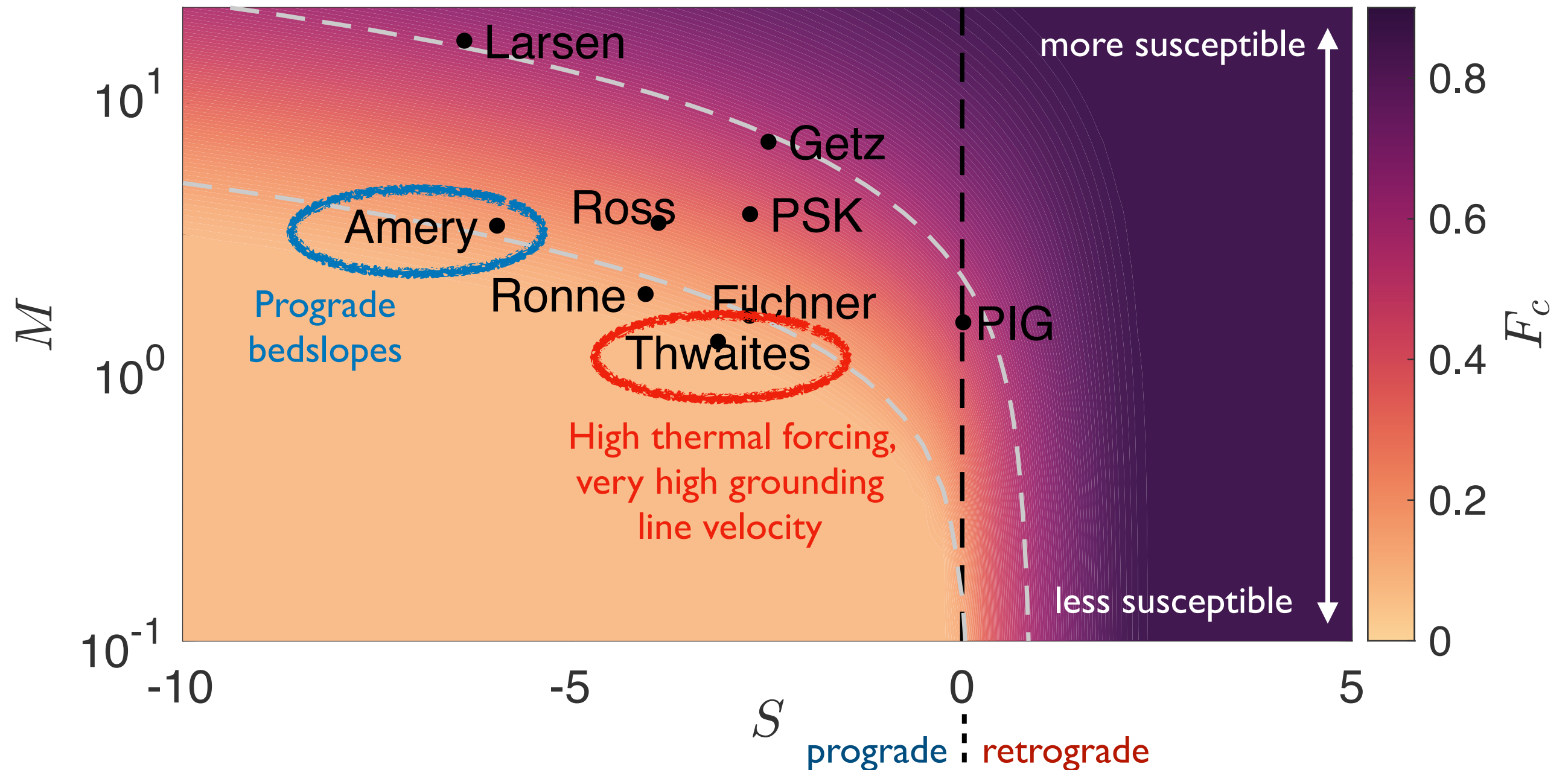
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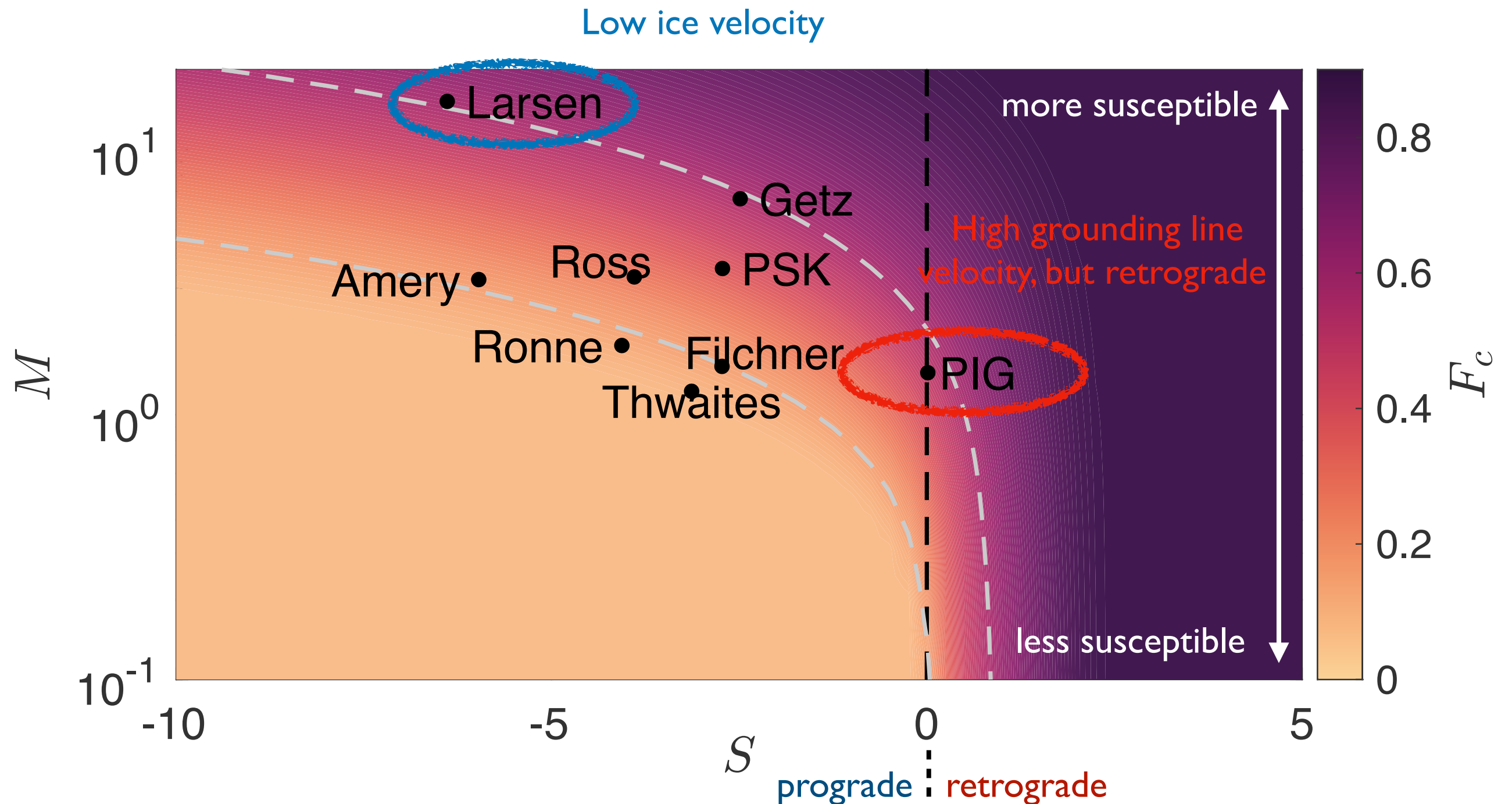
dimensionless bed slope

$$F = \frac{U_\infty}{\sqrt{g' H_\infty}}$$

upstream Froude number

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dimensionless melt





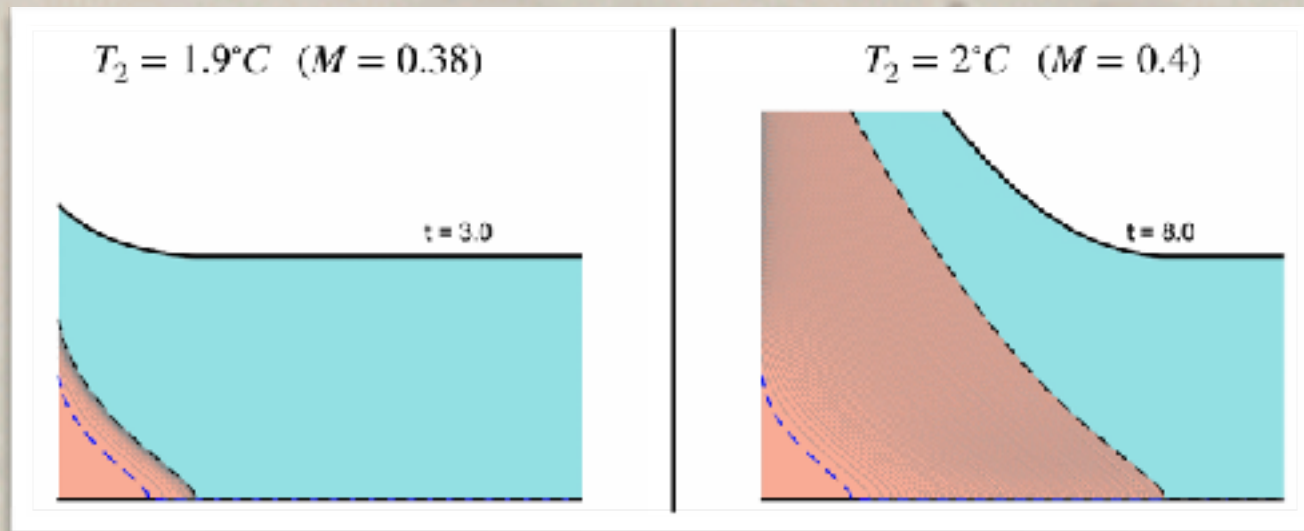
Melt feedback result in grounding zone tipping points



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Tipping point is 'generic'

Prograde slopes vulnerable, retrograde enhanced

Cold and warm cavity shelves susceptible

